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DIFFRACTION OF INTERNAL WAVES BY A CIRCULAR CYLINDER NEAR THE PYCNOCLINE

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Propagation of internal waves over a circular cylinder under the conditions of a continuous stratification characterized by the presence of a high-gradient density layer (the pycnocline) of finite thickness is studied. The dependences of the coefficient of wave propagation on the wavelength of the first-mode incident wave for various thicknesses of the pycnocline are obtained. In the diffraction of internal waves, substantial nonlinear effects are shown to occur, which result in the appearance of waves of double oscillation frequency compared to the frequency of the incident waves. The generation coefficient for these waves is found.

Diffraction of surface waves by a submerged circular cylinder is studied in sufficient detail. The absence of reflected waves in a fluid of finite depth is shown by Dean [1]. A theoretical description of this phenomenon within the framework of the dipole approximation was given by Tyvand [2]. In the general case, for a finite depth of the fluid, wave reflection occur. For certain discrete wavelengths of the incident waves, however, no reflection occurs [3, 4].

Linton and McIver [5] studied the diffraction of internal waves from a circular cylinder submerged into a two-layered fluid with a density jump at the interface. It was shown that if the circular cylinder is submerged into the lower, infinitely deep layer, no reflection of the surface or internal modes takes place. If the cylinder is placed into the upper layer of finite depth, the partial reflection of waves is observed.

Diffraction of internal waves by underwater obstacles under the conditions of continuous stratification shows a number of specific features brought about by excitation of the higher modes of the wave motion in the fluid [6]. A qualitative description of the phenomena observed is given, for example, in [7, 8].

The present work differs from all the previous studies in that a new method is used here to study the diffraction of internal waves. The essence of this method consists in simultaneous registration of the waves that pass over the cylinder and the waves that propagate in the absence of an obstacle.

The tests were carried out in a hydrodynamic $4.5 \times 0.2 \times 0.8$ m tray (Fig.1) which was filled with two layers of miscible fluids of different densities. First, the lower layer (the aqueous glycerin solution) of density $\rho_2 = 1.0095$ g/cm^3 and depth $h_2 = 30$ cm was poured onto the tray. Then, the upper layer (distilled water) of density $\rho_1 = 0.999$ g/cm^3 and a certain depth h_1 was poured using spreading drafts that floated on the free surface (marked with the triangle). Over the tray, a certain steady-state density distribution $\rho(z)$ was reached, which in the coordinate system attached to the free surface (with the z and x axes directed upward and in the direction of wave propagation, respectively) was approximated by the expression

$$\rho(z) = \rho_0 - \frac{\varepsilon\rho_1}{2}\tanh\frac{2(z+h_1)}{\delta}, \qquad \rho_0 = \frac{\rho_2+\rho_1}{2}, \qquad \varepsilon = \frac{\rho_2-\rho_1}{\rho_1}.$$

With time, only the characteristic thickness δ of the pycnocline varied owing to glycerin diffusion.

The density distribution $\rho(z)$, which was used to determine the parameter δ, was measured with probes 3 with a horizontally installed sensitive element [8]. Because of the smallness of the diffusion coefficient of glycerin, the thickness δ of the diffuse region varied insignificantly during one series of tests and was therefore

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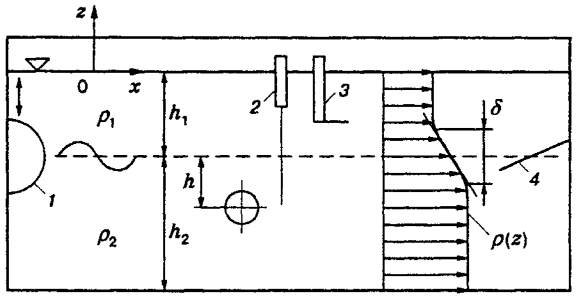

Fig. 1

assumed to be constant. The calibration measurements carried out prior and after the experiments showed that the increase in δ was within 0.2 cm.

The waves were generated by a wave generator 1 which had the shape of a semicylinder and executed sinusoidal oscillations along the end wall of the tray. The amplitude of the oscillations was $A = 0.6$ cm, their frequency ω being variable. At the other end edge of the tray, a wave suppressor 4 was provided, which had the shape of a flat plane inclined at an angle of 4° to the horizontal, with its lower end dipped by 4 cm below the conventional interface of the layers for which $\rho = \rho_0$.

Measurements of the coefficients of wave reflection and propagation for the surface waves over an obstacle are carried out, as a rule, by comparing the amplitudes of the waves measured at certain fixed points of the wave tray in the absence and presence of an obstacle. The procedure of [9] can also be used, which is based on an a priori assumption as to the shape of the free surface described in the far field by a superposition of the incident waves and a system of waves caused by diffraction. In the case of the internal waves in a continuously stratified fluid, the problem appeared to be more complicated owing to the necessity for exact reproduction of all parameters of the system, in particular, the thickness δ of the pycnocline. Therefore, in the present tests, the parameters of the waves behind the cylinder and those of the wave propagating in the absence of an obstacle were recorded simultaneously. To do this, a thin vertical partition (not shown in Fig.1) was provided that was installed along the tray and divided it into two equal parts. In one part of the tray, a rigidly fixed cylinder of diameter $d = 6$ cm was installed 170 cm from the wave generator, whose center was dipped by a distance h below the conventional interface. The vertical partition began 30 cm from the end wall of the tray.

The waves were measured by two wavemeters 2 of the resistive type [7] with vertically aligned sensitive elements. The wavemeters were spaced by the same distance from the wave generator in each part of the tray. One wavemeter measured the parameters of the undisturbed wave system, and the other the parameters of the waves that have passed over the circular cylinder. The distance from the plane at which the wavemeters were installed to the cylinder axis was 30 cm.

The wavemeters measured the electrical conductance of the medium situated between two vertical electrodes. The output signal $e(t)$ of the voltmeter installed at a certain point with the horizontal coordinate $x = x_0$ is related to the instantaneous value of density $\rho_*(x, z, t)$ by the dependence

$$e(t) = e_0 + e_1 \int_{z_1}^{z_2} \rho_*(x_0, z, t)\, dz,$$

where e_0 and e_1 are constants and z_1 and z_2 are the vertical coordinates of the lower and upper ends of the electrodes. Under the general assumptions of the theory of internal waves in a continuously stratified fluid [10, 11], the instantaneous density at a certain point can be represented in the form $\rho_* = \rho(z) + \hat{\rho}$, where $\rho(z)$ is the density distribution in the immobile fluid and $\hat{\rho}$ is the fluctuating density. Taking into account that at the point (x, z) and at a certain time a particle is situated that was initially present at the level $z - \zeta$ and retaining terms of the first-order smallness, we have $\hat{\rho} = \zeta(\partial\rho/\partial z)$. We represent the vertical displacement of the fluid particles in the wave in the form $\zeta = h(t)w(z)$, where $h(t)$ is a harmonic function of time and $w(z)$

is an amplitude function. With the assumptions made, the a.c. output signal of the wavemeter can be written in the form

$$\tilde{e} = e(t) - e_0 = e_1 h(t) \int_{z_1}^{z_2} \frac{d\rho}{dz} w(z)\, dz. \tag{1}$$

In the tests, the length of the probes was $(z_2 - z_1) > 2\delta$, and the following equalities were fulfilled with a high degree of precision at the end points of the interval of integration: $\rho(z_1) = \rho_2$, $\rho(z_2) = \rho_1$, and $\rho'(z_1) = \rho'(z_2) = 0$. Static calibration of the wavemeters was carried out using their vertical displacement by a preset distance c in the immobile fluid. The corresponding change in the output voltage $e_c = e_1 c(\rho_2 - \rho_1)$ results from (1) if we put $h(t) \equiv 1$ and $w(z) \equiv c$. This value was used to normalize the amplitude of the change of $\tilde{e}$. Thus, in measuring the wave intensity with the probes with vertical electrodes, the quantity

$$a = \frac{1}{\varepsilon \rho_1} \int_{z_1}^{z_2} \rho'(z) w(z)\, dz \tag{2}$$

is measured; this quantity has the dimension of length and represents the weighted $\rho'(z)$ mean amplitude of the vertical displacement of a fluid particle in the wave.

In the system of waves produced by the wave generator, the distribution $w(z)$ corresponded to the first internal mode of fluid wave motion. The measure of the intensity of these waves determined according to (2) is denoted by a_1. It is worth noting that in all the tests the amplitude of the waves was small ($2a_1 < 0.3$ cm).

During the passage of first-mode internal waves over an obstacle, generation of higher modes occurs. Moreover, because of nonlinear effects, the waves of the double frequency compared to the frequency of the incident waves (the second harmonics) are generated behind the cylinder. A spectral analysis of the output signal of the wavemeter, which measured the intensity of the waves behind the cylinder, allowed us to determine the amplitudes of the first and second harmonics. The corresponding measures of intensity are denoted by a_2 and a'.

The quantity $T = a_2/a_1$, which characterizes the ratio of the intensities of the disturbed and undisturbed wave systems, is adopted as the coefficient of propagation for the first-harmonic waves. It should be noted that the above definition is not a strict one. For surface waves, the notion of the coefficient of propagation is introduced for the far-field asymptotic behavior of these waves, and this squared coefficient gives the proportion between the energies of the passed and incident waves. For continuous stratification, the diffraction phenomena become more complex, since in this case many modes of the wave system become involved in the problem. Based on the known properties of the distribution of the vertical velocity of fluid particles for various modes of internal waves [11], one can easily see that the main contribution to the quantity a determined according to (2) is due to the uneven modes. Separation of the contribution of each of the uneven modes in the case where probes with vertical electrodes are used is difficult. Nevertheless, the quantity T is a sufficiently informative integral estimate, and it can be used to compare the experimental data with theoretical results. Doing it, one should take into account that the value of a depends on the phase shift between the oscillations of various modes.

For a preset frequency ω of the first-mode waves, their wavelength λ depends appreciably on the thickness δ of the pycnocline. In processing the experimental data, the following dispersion relationship was used [8, 11]:

$$\omega^2 = \varepsilon g k / (\coth k h_1 + \coth k h_2 + k\delta), \tag{3}$$

where $k = 2\pi/\lambda$ is the wave number. Direct measurements showed that the above relationship held in the tests with good accuracy. Figure 2 presents the dependence $\omega(k)$ for $\delta = 6.2$ cm [the solid curves and the points show the calculation results obtained using Eq. (3) and the experimental data, respectively].

Figure 3 shows the dependences of T on the dimensionless wave number $\bar{k} = kd$ for $\bar{h} = h/d = 1$, $\bar{h}_1 = h_1/d = 2.5$, and the values of the parameter $\bar{\delta} = \delta/d$ equal to 0.42, 0.55, and 0.72 (curves 1-3). For this combination of parameters, the cylinder is surrounded by a fluid of constant density ρ_2 for all values of $\bar{\delta}$. The dependences $T(\bar{k})$ lie close together for all $\bar{\delta}$ both qualitatively and quantitatively. In the experimental range of variation of $\bar{k}$, three minima and three maxima of T are seen. For short waves ($\bar{k} > 2$), the quantity $T \to 1$

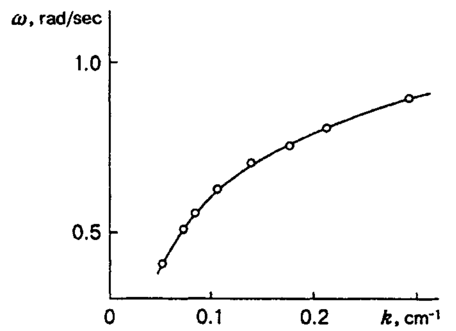

Fig. 2

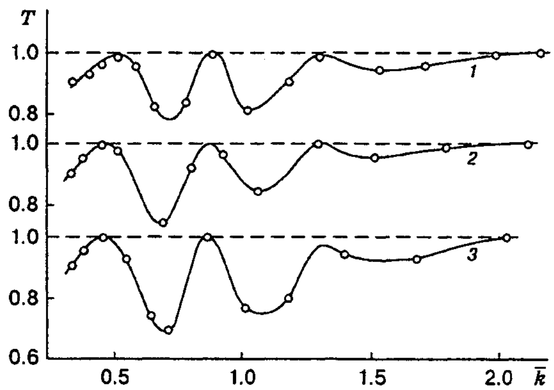

Fig. 3

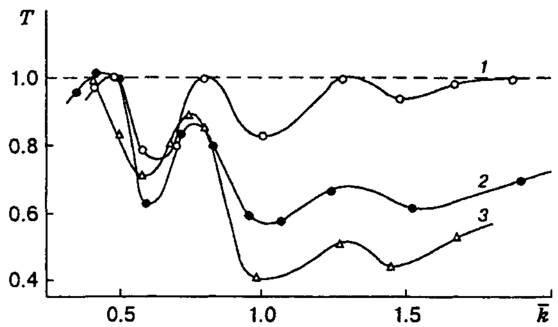

Fig. 5

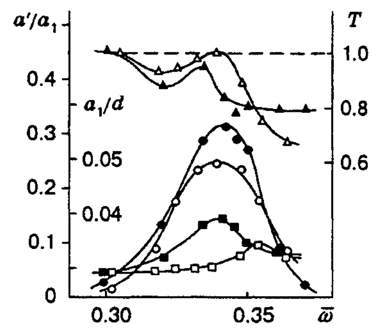

Fig. 6

(the waves pass over the body without being disturbed).

Figure 4 shows the data obtained for $\bar{h} = 0.75$, $\bar{h}_1 = 2.5$, and $\bar{\delta} = 0.25$, 0.38, 0.53, 0.72, 0.83, and 1.03 (curves 1–6, respectively). For this value of $\bar{h}$, the cylinder partially overlaps with the pycnocline. The value of T equals unity for long waves ($\bar{k} \approx 0.5$). The local maxima of T observed at high $\bar{k}$ are appreciably less than unity. However, the good coincidence between the positions of the characteristic points (the maxima and minima) in Figs. 3 and 4 is worth noting. As $\bar{\delta}$ increases, the value of T decreases markedly. In the short-wave range ($\bar{k} > 2$), T tends to increase, its value, however, being under unity. For short waves, the pycnocline acts as a wave guide, and the cylinder is an obstacle in it.

The effect of submergence of the cylinder into the pycnocline is illustrated by Fig. 5. The data were obtained for $\bar{h}_1 = 2.5$ and various values of $\bar{h}$, $\bar{\delta}$ being practically constant ($\bar{h} = 1$ and $\bar{\delta} = 0.58$ for curve 1; $\bar{h} = 0.75$ and $\bar{\delta} = 0.54$ for curve 2, and $\bar{h} = 0.6$, $\bar{\delta} = 0.58$ for curve 3). The value of T decreases appreciably with decreasing $\bar{h}$, and the decrease in T is more pronounced for short waves.

In wave diffraction by obstacles, the nonlinear effects give rise to waves with frequency twice that of the incident waves. For the case of waves on the free surface of a homogeneous fluid, this phenomenon was studied in sufficient detail in [9, 12, 13], where the dependence of the intensity of the second-harmonic waves was addressed. It was shown that under certain conditions the amplitude of the second harmonic can be as high as that of the second harmonic.

The nonlinear effects in the internal-wave diffraction have not been adequately studied. Some qualitative information on them was reported in [7, 8]. In contrast to surface waves, in a continuously stratified fluid there is a threshold value of $\omega = N_m/2$, where N_m is the maximum value of the Brunt–Väisälä frequency $N(z) = \sqrt{-(g/\rho)(\partial\rho/\partial z)}$. If the frequency of the incident waves $\omega > N_m/2$, then the second-harmonic surface waves do not originate [8, 14]. In this connection, the determination of the range of dimensionless frequencies

$\bar{\omega} = \omega/N_m$, in which the waves of frequency 2ω have the highest intensity, is of interest.

Figure 6 shows the dependences of the coefficient of generation of the second-harmonic waves a'/a_1 (circles), the value of T (triangles), and the dimensionless amplitude of the incident waves a_1/d (squares) on $\bar{\omega}$ for $\bar{\delta} = 0.42$ and 0.55 (open and filled points, respectively). The data were obtained for $\bar{h} = 0.75$ and $\bar{h}_1 = 2.5$. For surface waves of sufficiently small amplitude, the ratio a'/a_1 varies in proportion to a_1 [9, 13]. A similar dependence should also be expected for the case of internal waves. Since the amplitude and steepness of the incident waves in our experiments varied over a rather narrow range, the dependence of a'/a_1 on $\bar{\omega}$ gives a good indication of the range where the second-harmonic waves behind the cylinder are generated. It can be seen that the maxima of the dependences a'/a_1 on $\bar{\omega}$ for different $\bar{\delta}$ lie at one and the same value of the dimensionless frequency $\bar{\omega}_* = 0.34$ which corresponds to different values of $\bar{k}_*$ ($\bar{k}_* = 0.68$ for $\bar{\delta} = 0.42$ and $\bar{k}_* = 0.54$ for $\bar{\delta} = 0.55$). The constant value of $\bar{\omega}_*$ suggests the existence of a certain "resonant" regime of generation of the second-harmonic waves upon attaining a certain proportion between the wave number k' for the second-harmonic waves and the characteristic thickness δ of the pycnocline. For such short waves, the two fluid layers are infinitely deep. Using (3), it can be found that $\bar{\omega}_* = 0.34$ corresponds to $k'_*\delta = 2$.

It is worth noting that in a disturbed wave system that is combination of first-harmonic and second-harmonic waves, the local values of the wave steepness can far exceed the steepness of the incident waves. In our experiments, the maximum increase in the wave steepness was approximately threefold.

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